**Cosmological Dynamics**

So we have the RW metric and curvature,



(remember subscripts on variables mean deriviatives so act = ∂a/∂ct) Now let’s consider the stress-energy tensor. For a perfect fluid we have:



Imposing isotropy and homogeneity would seem to require no velocity – for instance I can’t think of how velocity could be distributed over the surface of a two-sphere which would be both isotropic and homogeneous for instance – and so we’d have = υ00 (no spatial part). From the normalization condition that 2 = -c2, we have υ0υ0 = -c2 --> g00υ0υ0 = -c2 --> = -(υ0)2 = -c2 --> υ0 = c. That would reduce to:



Now let’s work out Einstein’s equations for this metric.



We have:



and so,



which simplifies to Friedman’s Equations:



And recall k = 8πG/c4, ε0 = ρ0c2 is the rest energy density, and P the pressure. Might note that a nice equation that bypasses the need to know the curvature is obtained by adding the two equations:



These must be supplemented by the evolution equations of matter/energy itself. These follow from:



Filling into the equation we have:



where we recall that the 4-divergence of the metric tensor is zero, as shown in the Geometry file. So then working out the remainder, noting ε­0 and P can be functions of time, even if not space.



where we have used the relationship,



Now we’ll recall that our Christoffel symbol was:



which happens to be diagonal in the two lower indices for our case of interest. We’ll also note that gα0 = g00δα0. So,



This is taking longer than it should it seems. But now we have:



We can rewrite this as:



and finally,

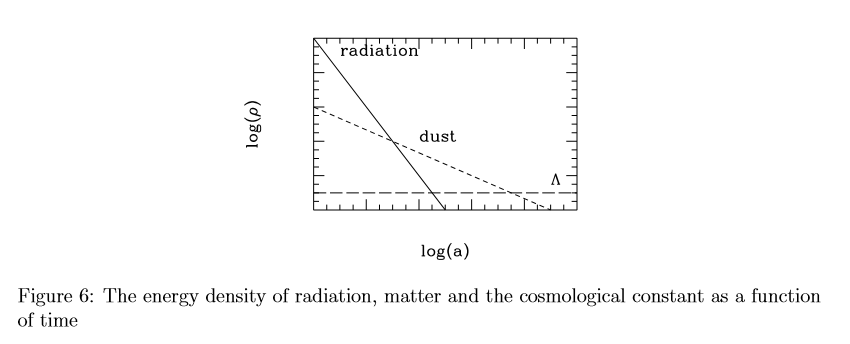


which has a nice straightforward interpretation that the rate of change of energy contained within our volume is equal to the rate of work done via pressure. Turns out that these three equations so far are not all linearly independent – only any two are. We would need one more equation to make this self-consistent – this would be the equation of state of our fluid, relating ε0 and P. There are a few options. For dust we would consider P = 0 of course. If we have a radiation, which we can think of as just energetic massless particles, for which the previous analysis still applies, then we can use the results we obtained/will obtain in the thermodynamics folder, that P = ε0/3. So,



**Predominance of various energy source terms over time**

During the early early universe it is presumed the cosmological constant dominated over the other two, causing rapid inflation, but then sort of ‘expended itself’ to its present constant value. Shortly thereafter at the initial super high temperatures, radiation was the predominant energy source. As the universe expanded, the radiation energy density fell ρrad ~ 1/a4; 1/a3 due to energy conservation and 1/a due to redshift. As the universe cooled, matter began to form and predominate as an energy source, but also decayed ρmatt ~ 1/a3 due to energy conservation and the universe’s expansion. Finally, in the present era, Λ has again become predominant.



Present estimates are that dark energy supplies 70% of the energy content of the universe, dark matter 25%, and ordinary matter 5%. Radiation comprises about 0.1%. FYI, the presence of dark matter can be inferred as follows. Suppose we have at the center of the galaxy a very massive center, which, given the observed density of objects is the case. Then the speed of objects rotating about the center ought to be, according to N2L, and F = Gm1m2/r2,



But it is found that v is roughly constant with radius, indicating that M must increase linearly with r. But no such matter is observed. Thus it is called dark matter. Options are WIMPS (weakly interacting massive particles), and MACHOS (massive compact halo objects).

**Curvature**

The first metric equation, say, can tell us something about the curvature of the universe. Solving for κ we have:



So if we know the energy density, cosmological constant, and Hubble parameter, we can determine the sign of κ. The former must include dark matter in addition to the regular ‘baryonic’ matter that we can directly observer. The amount of dark matter in the universe can be inferred from observations of galaxial rotations. The cosmological constant can be inferred from something. And estimates of H have been made fairly precise now. Altogether, it seems that κ is pretty much zero. Since this is constant, whatever we get now, should be what we have always had. Other more direct estimates of κ can be made, bypassing this route. And these confirm the nill result, which incidentally, provides more indirect evidence of the presence of Λ.

**Stage 1: Inflation (t < 10-14s)**

It is supposed that, to explain how matter densities, like He, are basically uniform throughout the observable universe, along with other uniformities, that our patch of universe expanded rapidly from a small patch of universe (and anything can be taken to be uniform over a sufficiently small size) in a process known as inflation. During this process, the cosmological constant dominated all other inputs to determine the evolution of the universe. The Friedman equations in this case reduce to, assuming zero curvature:



The first equation has the solution:



You can verify that this also solves the second equation. We clearly have exponential growth. Λ is supposed to have been large enough to rapidly expand the universe from the Plank scale ~ 10-35m to something on the order of what we see today. Inflationary models of the universe are capable of explaining fluctuations in the cosmic microwave background.

**Stage 2: Radiation Dominated Universe (10-14s < t < 104 yrs)**

After inflation, the evolution of the universe is believed to have been dominated by radiation. This is the start of the Big Bang era I think. Perhaps the dark energy was ‘used up’ in some sense. The Friedman equations + stress-energy equations governing the evolution of the universe would therefore be, again presuming zero curvature:

The stress-energy equations give us:



Filling this into the top metric equation we have:



Then putting this back into our stress-energy equation we have:



So all total we have:



We can see that ε0(t) ~ 1/t2, and a(t) ~ √t. These imply ε0 ~ 1/a4, which can be understood from conservation of energy grounds. ε0 ~ 1/a3 from energy conservation. But then since it consists of photons and they are redshifted by the expanding universe, there is an extra factor of 1/a. So it goes as 1/a4. Note that this result imply a universe whose expansion is decelerating, as ~ 1/√t. The expansion of the universe also implies cooling. Not because the universe pushes against anything per se´, but because as the universe expands, the photons expand with it and thereby lose energy. So the temperature of the universe, at least back then, went as 1/a(t). Using



from statistical mechanics, we could calculate the temperature of the universe as a function of time, during this epoch. We can calculate the Hubble parameter in this model:



if we solve for time we get:



If the last term is negligible, then we can estimate the length of this phase of the universe as just,



But we’d need to know H at that time. Anyway, while radiation dominates, it isn’t the only thing. A quark-gluon plasma forms at about 10-9s and lasts up until about 10-6s after the Big Bang. As the universe cools, matter and antimatter begins to form (and I assume whatever dark matter is began to form too). Protons, neutrons, and their anti’s begin to form and quickly matter comes to dominate over antimatter. These fuse into heavy element nuclei, releasing a lot of photons in process. Simple ionized atoms also begin to form. So we have a plasma of charged particles in thermal equilibrium (presumably with Plank distribution) with the ambient radiation. But photons are trapped within the plasma.

**Stage 3: Matter Dominated Universe (104 yrs < t < 1010 yrs)**

As universe cooled, neutral matter begins to be formed, and begins to dominate radiation as the primary energy source. Matter, or dust, is characterized by zero pressure. So our equations come to, again presuming zero curvature:

The stress-energy equations give us:



Filling this into the metric equation we have:



and so altogether,



We can calculate the Hubble parameter in this model:



if we solve for time we get:



If the last term is negligible, then we can estimate the length of this phase of the universe as just,



As the temperature of the universe futher cooled, neutral hydrogen atoms could form out of the soup. This is the start of the Recombination era (positive nuclei and negative electrons combine). Additionally, the neutral atoms can now drop from their excited states into their ground state, releasing more photons (this is called decoupling). Once neutral matter predominated, around 370,000 years after the Big Bang, this allowed all the radiation, formerly trapped within the plasma by scattering, to escape and permeate the rest of the universe. Originally, this radiation would’ve been in the visible spectrum and appeared a brilliant orange hue. This relic radiation, which continues to redshift as the universe expands is the cosmic microwave background. Now of course, it has redshifted to way below the visible spectrum. When it dropped below the visible spectrum the so-called Dark Ages began. The redshifting would would mean that the radiation keeps its Plank distribution but with a decreasing temperature T ~ 1/a. Calculations of what that temperature should be have matched experimental observations rather precisely, giving dramatic confirmation of Einstein’s equations applied to the universe. Conversely, we can use this same fact to go backwards and calculate the temperature of the universe as a function of time. Eventually, after about 108 years, clouds of neutral hydrogen gravitationally coalesced into stars, and gave light to the universe again. The highly energetic photons released permeate the universe with ionizing radiation, leading to the error of Reionization, at about 109 years.

**Stage 4: Dark Energy Dominated Universe (1010 yrs < t)**

As the universe continues to expand, the matter density begins to drop of course. At around 10 billion years, it appears that the matter-energy density dropped below the relic dark energy density left over since inflation. And so once again, we’re in a dark energy dominated universe. And our solution is:



And the universe is beginning to expand outward at an accelerating rate. We can calculate the Hubble parameter in this model:



So it would seem this is one way to calculate Λ.